

Low-Complexity Joint Channel Estimation and List Decoding of Short Codes

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Abstract—A pilot-assisted transmission (PAT) scheme is proposed for short blocklengths, where the pilots are used only to derive an initial channel estimate for the list construction step. The final decision of the message is obtained by applying a non-coherent decoding metric to the codewords composing the list. This allows one to use very few pilots, thus reducing the channel estimation overhead. The method is applied to an ordered statistics decoder for communication over a Rayleigh block-fading channel. Gains of up to 1.2 dB as compared to traditional PAT schemes are demonstrated for short codes with QPSK signaling. The approach can be generalized to other list decoders, e.g., to list decoding of polar codes.

I. INTRODUCTION

The interest in designing wireless communication systems with short information blocks, up to a few tens of bits, has been increasing recently due to the rise of applications characterized by strict latency constraints [1]. As a consequence, the fundamental limits of communications for finite-length messages have received renewed attention (see, e.g., [2]–[5]). Code designs [6]–[8] and sophisticated decoding algorithms [9], [10] targeting near-optimal performance in the moderate- and short-length regimes have been proposed. Using such methods, it is possible to operate close to the finite length bounds (see, e.g., [11] for a comparison of short code constructions and finite length bounds). While most of the attention has been focused on communication over additive white Gaussian noise (AWGN) channels, it is also interesting to communicate with short packets over a fading channel with no channel state information (CSI) available at the transmitter and receiver. In fact, classic pilot-assisted transmission (PAT) methods [12] turn out to be highly sub-optimal when short blocks are used [13]. The rates achievable over fading channels when the CSI is not available a priori has been investigated in [14]–[16] for a fixed blocklength and error probability. Bounds on the error probabilities are provided in [17] not only for non-coherent transmission but also for PAT strategies.

We extend the work of [17] by introducing a PAT scheme with very few pilot symbols. The pilot symbols are used to obtain a (potentially rough) channel estimate, which is then employed by a list decoder to explore the neighborhood of

the channel observation, i.e., to construct a list of candidate codewords that achieve a large likelihood given the available channel estimate. The final decision is then performed by selecting the codeword in the list according to a non-coherent decoding metric. The role of the pilot symbols is thus to enable the construction of a good list—a task that is less challenging than deriving directly a decision on the transmitted codeword. This enables one to allocate very few pilots, hence reducing the pilot overhead. The principle can be applied to list decoders in general and to various slow fading channels. As an example, we apply the method to the ordered-statistics decoder (OSD) of [9] over a single-input single-output (SISO) Rayleigh block-fading channel.

The paper is organized as follows. In Section II, we present the system model and various decoding criteria, and we discuss the complexity of non-coherent decoding metrics. Motivated by the complexity argument, we review classic PAT approaches in Section III. A list decoding method is presented in Section IV. Finite-length performance bounds are provided in Section V, followed by numerical results and conclusions in Sections VI and VII, respectively.

II. PRELIMINARIES

We use capital letters, e.g., X , for random variables (RVs) and their lower case counterparts, e.g., x , for their realizations. We denote the random vectors via capital bold letters, e.g., $\mathbf{X} = [X_1, X_2, \dots, X_n]$, and their vector realizations via the lower case counterparts, e.g., $\mathbf{x} = [x_1, x_2, \dots, x_n]$. As an exception, \mathbf{I}_a refers to the $a \times a$ identity matrix. The probability mass function of the discrete RV X is denoted as P_X , whereas the probability density function of the continuous RV X is denoted as p_X . We use $\|\cdot\|$ for the l^2 -norm, $\langle \cdot, \cdot \rangle$ for the inner product of two vectors, $\ln(\cdot)$ for the natural logarithm, and $\mathbb{E}[\cdot]$ for the expectation. We write $\mathcal{CN}(\mu, \sigma^2)$ to denote a complex Gaussian RV with mean μ and variance σ^2 .

A. System Model

We consider a SISO Rayleigh block-fading channel, i.e., the random fading coefficient is constant for n_c channel uses and changes independently across ℓ coherence blocks, which are also called diversity branches. Therefore, the packet size is $n = \ell n_c$. Such a setup is relevant for orthogonal frequency-division multiplexing (OFDM) systems, e.g., LTE and 5G (see

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[16]). The input-output relationship of the channel for the i th coherence block is

$$\mathbf{y}_i = h_i \mathbf{x}_i + \mathbf{n}_i, \quad i = 1, \dots, \ell \quad (1)$$

where $\mathbf{x}_i \in \mathcal{X}^{n_c}$ and $\mathbf{y}_i \in \mathbb{C}^{n_c}$ denote the transmitted and received vectors, h_i is the realization of the channel coefficient, which is distributed as $H_i \sim \mathcal{CN}(0, 1)$ and \mathbf{n}_i is the corresponding AWGN term, which is distributed as $\mathbf{N}_i \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{n_c})$. The mutually independent RVs H_i and \mathbf{N}_i are assumed to be independent over i . We will focus on quadrature phase shift keying (QPSK) signalling where energy per symbol is normalized to 1.

B. Decoding with Perfect CSI

If the channel coefficients are known to the receiver, the (coherent) maximum likelihood (ML) decoding rule is

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{C}} p_{\mathbf{Y}|\mathbf{X}, \mathbf{H}}(\mathbf{y}|\mathbf{x}, \mathbf{h}) \quad (2)$$

$$= \arg \min_{\mathbf{x} \in \mathcal{C}} \sum_{i=1}^{\ell} \|\mathbf{y}_i - h_i \mathbf{x}_i\|^2 \quad (3)$$

where \mathcal{C} is the set of transmitted signal vectors induced by the chosen channel code and modulation. When $\|\mathbf{x}_i\|$ is constant across codewords and blocks, we have

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{C}} \sum_{i=1}^{\ell} \Re\{\langle \mathbf{y}_i, h_i \mathbf{x}_i \rangle\} \quad (4)$$

which is the case, for instance, if the modulation is QPSK.

C. Decoding without CSI—Pilot-Assisted Channel Estimation

The idealized setting described in Section II-B is often approximated by including pilot symbols in the transmitted sequence, which are used to obtain an estimate of the channel coefficients. This estimate $\hat{\mathbf{h}}$ is treated as ideal by a mismatched decoder, yielding

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{C}} p_{\mathbf{Y}|\mathbf{X}, \mathbf{H}}(\mathbf{y}|\mathbf{x}, \hat{\mathbf{h}}) \quad (5)$$

$$= \arg \min_{\mathbf{x} \in \mathcal{C}} \sum_{i=1}^{\ell} \|\mathbf{y}_i - \hat{h}_i \mathbf{x}_i\|^2. \quad (6)$$

For QPSK, this reduces to

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{C}} \sum_{i=1}^{\ell} \Re\{\langle \mathbf{y}_i, \hat{h}_i \mathbf{x}_i \rangle\}. \quad (7)$$

D. Decoding without CSI—Blind Approach

Assume next that the decoder does not have access to the channel coefficients, and that no pilots are embedded in the transmitted sequence. In this case, we distinguish between two possibilities: *i*) The decoder does not possess information on the distribution of the channel coefficients and *ii*) the decoder knows the channel coefficients' distribution. In case *i*), the problem can be tackled, for instance, by designing a generalized likelihood ratio test (GLRT) as in [18] yielding

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{C}} \sup_{\mathbf{h}} p_{\mathbf{Y}|\mathbf{X}, \mathbf{H}}(\mathbf{y}|\mathbf{x}, \mathbf{h}) \quad (8)$$

$$= \arg \min_{\mathbf{x} \in \mathcal{C}} \sum_{i=1}^{\ell} \inf_{h_i} \|\mathbf{y}_i - h_i \mathbf{x}_i\|^2 \quad (9)$$

$$= \arg \max_{\mathbf{x} \in \mathcal{C}} \sum_{i=1}^{\ell} \frac{|\langle \mathbf{y}_i, \mathbf{x}_i \rangle|^2}{\|\mathbf{x}_i\|^2}. \quad (10)$$

The last step follows because the ML channel estimate is $\hat{h}_i = \langle \mathbf{y}_i, \mathbf{x}_i \rangle / \|\mathbf{x}_i\|^2$. For QPSK, (10) reduces to

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{C}} \sum_{i=1}^{\ell} |\langle \mathbf{y}_i, \mathbf{x}_i \rangle|^2. \quad (11)$$

In case *ii*), the non-coherent ML estimate is

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{C}} \prod_{i=1}^{\ell} \mathbb{E}[p_{\mathbf{Y}|\mathbf{X}, \mathbf{H}}(\mathbf{y}_i|\mathbf{x}_i, H)] \quad (12)$$

$$= \arg \max_{\mathbf{x} \in \mathcal{C}} \sum_{i=1}^{\ell} \frac{|\langle \mathbf{y}_i, \mathbf{x}_i \rangle|^2}{\sigma^2(\sigma^2 + \|\mathbf{x}_i\|^2)} - \ln \left(1 + \frac{\|\mathbf{x}_i\|^2}{\sigma^2} \right) \quad (13)$$

where (13) follows because the conditional received vector \mathbf{y}_i per coherence block given the transmitted sequence \mathbf{x}_i is complex Gaussian with mean $\mathbb{E}[\mathbf{y}_i|\mathbf{x}_i] = \mathbf{0}$ and covariance $\mathbb{E}[\mathbf{y}_i^H \mathbf{y}_i|\mathbf{x}_i] = \sigma^2 \mathbf{I}_{n_c} + \mathbf{x}_i^H \mathbf{x}_i$. For QPSK, we get

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{C}} \sum_{i=1}^{\ell} |\langle \mathbf{y}_i, \mathbf{x}_i \rangle|^2. \quad (14)$$

Note that (11) coincides with (14) under the assumption that the signals in \mathcal{C} have the same energy over each coherence block, e.g., for QPSK signaling.

E. On the Complexity of Non-Coherent Decoding

By inspecting (11) and (14), we see that the decoding metric does not admit a trivial factorization, hindering the use of efficient maximum metric decoders (such as Viterbi decoding over the code trellis) and of any decoding algorithm that relies on the factorization of the channel likelihood (such as belief propagation decoding for turbo/low-density parity-check codes or successive cancellation decoding of polar codes). A pragmatic solution to this problem is to embed a small number of pilots in the transmitted sequence, which are then used to bootstrap iterative decoding and channel estimation algorithms [19]–[21]

In the following, we first discuss how the iterative decoding and channel estimation approach can be applied to OSD (Section III). Then, we show that list decoders in general, and OSD in particular, allow for an alternative approach to non-coherent decoding (Section IV) which yields simultaneously a gain in error correction capability and a reduction in decoding complexity.

The general framework for the algorithms presented in the following sections relies on the PAT approach of Section II-C. More specifically, we embed n_p pilot symbols into each coherence block. For the i th coherence block, the vector of pilot symbols is denoted by \mathbf{x}_i^p . The pilots are followed by $n_c - n_p$ coded symbols, denoted by \mathbf{x}_i^d . The corresponding

channel outputs are denoted by \mathbf{y}_i^p and \mathbf{y}_i^d , respectively. The rate in terms of bits per channel use (bpcu) is

$$R = \frac{k}{\ell n_c} \quad (15)$$

where k is the number of information bits encoded by \mathcal{C} . The rate of the code \mathcal{C} is instead denoted by

$$R_0 = \frac{k}{\ell(n_c - n_p)}. \quad (16)$$

As a result, for a fixed rate R and a fixed blocklength ℓn_c , a large number of pilots comes at the cost of an increase in the code rate R_0 , and thus a reduction of the error correction capability. This yields a trade-off between resources allocated to channel estimation and error correction (see [13]).

III. CLASSIC APPROACHES

We illustrate two ways of using OSD, which will be taken as references for the novel algorithm presented in Section IV. The first approach is a plain application of the PAT scheme sketched in Section II-C. The second approach iterates pilot-aided channel estimation and OSD by means of the expectation maximization (EM) algorithm. Upon observing the channel output, both approaches use the pilot symbols in each coherence block to perform an ML estimation of the corresponding channel coefficient, i.e., we have

$$\hat{h}_i = \frac{\langle \mathbf{y}_i^p, \mathbf{x}_i^p \rangle}{\|\mathbf{x}_i^p\|}, \quad i = 1, \dots, \ell. \quad (17)$$

A. Pragmatic Pilot-Assisted Ordered-Statistics Decoder

The channel estimates (17) are treated as perfect and the bit-wise log-likelihood ratios (LLRs) based on the mismatched likelihoods $p_{\mathbf{Y}^d|\mathbf{X}^d, H}(\mathbf{y}_i^d|\mathbf{x}_i^d, \hat{h}_i)$, with $i = 1, \dots, \ell$, are fed to the OSD. After constructing the list \mathcal{L} , one applies the metric in (7) to the codewords in the list, yielding

$$\hat{\mathbf{x}}^d = \arg \max_{\mathbf{x}^d \in \mathcal{L}} \sum_{i=1}^{\ell} \Re\{\langle \mathbf{y}_i^d, \hat{h}_i \mathbf{x}_i^d \rangle\}. \quad (18)$$

B. Iterative Channel Estimation and Ordered-Statistics Decoding via Expectation-Maximization

We reduce the number of pilots (and hence allow for the use of a lower-rate code) by iterating channel estimation and channel decoding [19], [20]. In the following, we describe how the EM algorithm [22] can be used for this purpose, in combination with OSD. The algorithm works as follows:

1. Initialize $\hat{h}_i^{(0)}$ as in (17) for $i = 1, \dots, \ell$, and construct the list $\mathcal{L}^{(0)}$ using the channel estimates.
2. At iteration j , we construct the list $\mathcal{L}^{(j)}$ using the updated channel estimates $\hat{h}_i^{(j)}$. Then, we have
 - a. Expectation step:

$$Q(h_i, \hat{\mathbf{h}}^{(j-1)}) = \sum_{\mathbf{x}^d \in \mathcal{L}^{(j-1)}} -P_{\mathbf{X}^d|\mathbf{Y}^d, H}(\mathbf{x}^d|\mathbf{y}^d, \hat{\mathbf{h}}^{(j-1)}) \times \|\mathbf{y}_i^d - h_i \mathbf{x}_i^d\|^2 \quad (19)$$

where we approximate $P_{\mathbf{X}^d|\mathbf{Y}^d, H}(\mathbf{x}^d|\mathbf{y}^d, \hat{\mathbf{h}}^{(j-1)})$ as

$$\frac{p_{\mathbf{Y}^d|\mathbf{X}^d, H}(\mathbf{y}^d|\mathbf{x}^d, \hat{\mathbf{h}}^{(j-1)})}{\sum_{\tilde{\mathbf{x}}^d \in \mathcal{L}^{(j-1)}} p_{\mathbf{Y}^d|\mathbf{X}^d, H}(\mathbf{y}^d|\tilde{\mathbf{x}}^d, \hat{\mathbf{h}}^{(j-1)})}. \quad (20)$$

- b. Maximization step:

$$\hat{h}_i^{(j)} = \arg \max_{h_i} Q(h_i, \hat{\mathbf{h}}^{(j-1)}). \quad (21)$$

After performing Step 2 for a predetermined number m of iterations, the final decision is obtained as in (18) by replacing $\hat{\mathbf{h}}$ and \mathcal{L} by $\hat{\mathbf{h}}^{(m)}$ and $\mathcal{L}^{(m)}$, respectively.

IV. ORDERED-STATISTICS DECODING WITH IN-LIST GLRT

We use the channel estimate to form the list \mathcal{L} of codewords via the OSD procedure as in Section III-A. Then, each codeword in the list is modified by re-inserting the pilot field, which yields a modified list \mathcal{L}' . The final codeword is picked among \mathcal{L}' according to the GLRT rule given by (11), i.e., we choose

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{L}'} \sum_{i=1}^{\ell} |\langle \mathbf{y}_i, \mathbf{x}_i \rangle|^2 \quad (22)$$

$$= \arg \max_{\mathbf{x} \in \mathcal{L}'} \sum_{i=1}^{\ell} \Re\{\langle \mathbf{y}_i^d, \hat{h}_i \mathbf{x}_i^d \rangle\} + \frac{1}{2n_p} |\langle \mathbf{y}_i^p, \mathbf{x}_i^p \rangle|^2. \quad (23)$$

This metric lends itself to an alternative interpretation. Suppose that the distribution of the channel coefficient for the i th coherence block is a complex Gaussian distribution with mean \hat{h}_i given in (17) and variance $\frac{2\sigma^2}{\|\mathbf{x}_i^p\|^2}$, i.e., $H_i \sim \mathcal{CN}(\hat{h}_i, \frac{2\sigma^2}{\|\mathbf{x}_i^p\|^2})$. Then, similar to (12), we obtain

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{L}'} \prod_{i=1}^{\ell} \mathbb{E}[p_{\mathbf{Y}^d|\mathbf{X}^d, H_i}(\mathbf{y}_i^d|\mathbf{x}_i^d, H_i)] \quad (24)$$

$$= \arg \max_{\mathbf{x} \in \mathcal{L}'} \sum_{i=1}^{\ell} \frac{\|\mathbf{x}_i^d\|^2 + |\langle \mathbf{y}_i^d, \mathbf{x}_i^d \rangle|^2 + 2\|\mathbf{x}_i^p\|^2 \Re\{\langle \mathbf{y}_i^d, \hat{h}_i \mathbf{x}_i^d \rangle\}}{\|\mathbf{x}_i^p\|^2 + \|\mathbf{x}_i^d\|^2} - |\hat{h}_i|^2 \|\mathbf{x}_i^d\|^2 + \sigma^2 \ln \left(\frac{\|\mathbf{x}_i^p\|^2}{\|\mathbf{x}_i^p\|^2 + \|\mathbf{x}_i^d\|^2} \right) \quad (25)$$

where \mathcal{L}' is the modified channel code obtained by re-inserting the pilot symbols to each codeword. By assuming QPSK (which implies $\|\mathbf{x}_i^p\|^2 = n_p$), we recover (23). Note that the decoding metric has two contributions: A first term that resembles a coherent metric based on the estimate $\hat{\mathbf{h}}$, and a second term that is related to the non-coherent correlation. The second term is weighted by the inverse of the number of pilots; hence it becomes negligible when n_p is large (i.e., when the channel estimate is reliable).

V. FINITE-BLOCKLENGTH BOUNDS

We review the converse and achievability bounds on the average error probability based on finite-blocklength information theory that will be used to benchmark the coding schemes introduced in the previous section. The converse bound is

based on the metaconverse theorem in [5, Thm. 28] and the achievability bounds are based on the random-coding union bound with parameter s (RCUs) [23, Thm. 1].

Let $q : \mathbb{C}^{n_c} \times \mathbb{C}^{n_c} \rightarrow \mathbb{R}^+$ be an arbitrary block-wise decoding metric and let $(\bar{\mathbf{X}}_i, \mathbf{X}_i, \mathbf{Y}_i) \sim p_{\mathbf{X}}(\bar{\mathbf{x}}_i)p_{\mathbf{X}}(\mathbf{x}_i)p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}_i|\mathbf{x}_i)$, $i = 1, \dots, \ell$, be independent across coherence blocks. The generalized information density is defined as

$$v_s(\mathbf{x}_i, \mathbf{y}_i) \triangleq \ln \frac{q(\mathbf{x}_i, \mathbf{y}_i)^s}{\mathbb{E}[q(\bar{\mathbf{X}}_i, \mathbf{y}_i)^s]} \quad (26)$$

where $s \geq 0$. The RCUs achievability bound states that, for a given rate R , the average error probability is upper-bounded as

$$\epsilon \leq \epsilon_{\text{rcus}} \quad (27)$$

$$\triangleq \inf_{s \geq 0} \mathbb{E} \left[e^{-[\sum_{i=1}^{\ell} v_s(\mathbf{X}_i, \mathbf{Y}_i) - \ln(2^{Rn_c\ell} - 1)]^+} \right]. \quad (28)$$

We evaluate the bound in (28) for the following combinations of input distributions and decoding metrics:

- i) Input symbols uniformly distributed on a shell in \mathbb{C}^{n_c} , and ML decoding, i.e., $q(\mathbf{x}_i, \mathbf{y}_i) = p_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}_i|\mathbf{x}_i)$;
- ii) a pilot-assisted scheme as in Section II-E with the $n_c - n_p$ data symbols uniformly distributed on a shell in $\mathbb{C}^{n_c - n_p}$ and ML decoding, i.e., $q(\mathbf{x}_i, \mathbf{y}_i) = p_{\mathbf{Y}^d|\mathbf{X}^d, \hat{\mathbf{H}}}(\mathbf{y}_i^d|\mathbf{x}_i^d, \hat{\mathbf{h}}_i)$;
- iii) input distribution as in ii), and scaled nearest neighbor decoding, i.e., $q(\mathbf{x}_i, \mathbf{y}_i) = \exp(-\|\mathbf{y}_i^d - \hat{\mathbf{h}}_i \mathbf{x}_i^d\|^2)$.

See [17, Sec. III.A-III.D] for additional details on how to evaluate (28) for each of these cases.

Next, we state the converse bound, which is based on the metaconverse theorem. For a given average error probability ϵ , the maximum code rate is upper-bounded as

$$R \leq R_{\text{mc}}(\epsilon) \quad (29)$$

$$\triangleq \inf_{\lambda \geq 0} \frac{1}{\ell n_c} \left(\lambda - \ln \left[\mathbb{P} \left[\sum_{i=1}^{\ell} v_1(\mathbf{X}_i, \mathbf{Y}_i) \leq \lambda \right] - \epsilon \right]^+ \right). \quad (30)$$

For a given rate R , a lower bound on ϵ , denoted as ϵ_{mc} , can be obtained from (30) by finding the ϵ_{mc} for which $R_{\text{mc}}(\epsilon_{\text{mc}}) = R$. For more details on this converse bound, the reader is referred to [17, Sec. III.E].

VI. NUMERICAL RESULTS

We present next an example of the performance achieved by the decoder proposed in Section IV. The results are obtained by Monte Carlo simulations and are provided in terms of block error rate (BLER) vs. signal-to-noise ratio (SNR) with the SNR expressed as E_s/N_0 , where E_s is the expected energy per symbol and N_0 the single-sided noise power spectral density. The results are compared with the bounds of Section V. We consider a Rayleigh block-fading channel with 4 diversity branches. Each branch consists of 13 channel uses, which results in 52 channel uses per message. For the simulations, we considered the case where $k = 32$ information bits are transmitted within each codeword, yielding a rate $R = 32/52 \approx 0.62$ bits per channel use. The symbols are

taken from a QPSK constellation. A (96, 32) quasi-cyclic code is used to transmit and a suitable number of codeword bits is punctured to accommodate the pilot symbols within the 52 channel uses. The code is obtained by tail-biting termination of a rate-1/3 non-systematic convolutional code with a memory 17 and generators [552137, 614671, 772233] [24, Table 10.14]. The minimum distance of the quasi-cyclic code is upper-bounded by the free distance of the underlying convolutional code, which is 32. After encoding, a pseudo-random interleaver is applied to the codeword bits. Then puncturing adapts the blocklength to the number of channel uses available after pilot insertion. The OSD order is set to 3, which provides a reasonable trade-off between performance and decoding complexity. With this choice, OSD builds a list \mathcal{L} of $\sum_{i=0}^3 \binom{k}{i} = 5489$ candidate codewords.

In Fig. 1, we compare the performance of the proposed decoder to the performance of the two baseline decoders described in Section III for different numbers of pilot symbols ($n_p \in \{1, 2, 3\}$) per coherence block. For the iterative EM-based OSD, we set the number of iteration to $m = 1$. For the tested cases, the gain achieved by the proposed decoder is up to 1.2 dB as compared to the simple pilot-aided OSD of Section III-A. The performance of the iterative EM-based OSD is only marginally better than the one obtained by the simple pilot-aided OSD. Remarkably, the proposed approach performs close to the RCUs for PAT and ML decoding except for $n_p = 1$. In the simulated setting, the proposed approach provides the best performance with $n_p = 2$, with a slight degradation visible when $n_p = 3$.

VII. CONCLUSIONS

We proposed a novel decoding method over fading channels with no CSI at the transmitter/receiver, which leverages an initial (rough) pilot-assisted channel estimate to construct a list, and then performs the final decision by applying a non-coherent decoding metric to the list elements. The approach can be applied to codes that admit list decoding. We demonstrated its application to OSD, and showed that, in the short blocklength regime, it is possible to operate close to tight random coding achievability bounds.

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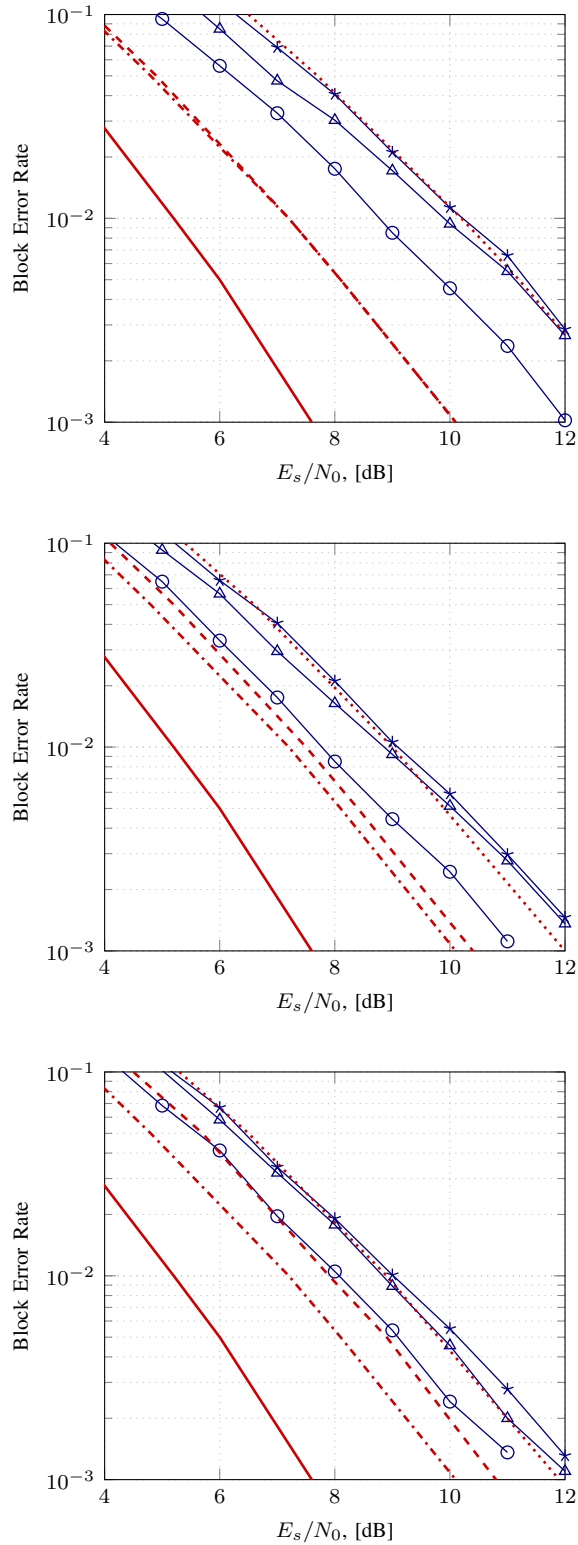


Fig. 1: BLER vs. SNR for the proposed scheme (\circ) with $n_p = 1$ (top), $n_p = 2$ (middle) and $n_p = 3$ (bottom). Finite length performance bounds given by the converse bound of (30) (—), the achievability of (28) for a non-coherent setup with ML decoding (·····), for PAT under ML decoding (---) and for PAT under scaled nearest neighbor decoding (·····). The performance of the pragmatic PAT OSD scheme of Section III-A (\star) and the performance of the EM-based approach (\triangle) is provided as a reference.

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